Egocentric Mapping of Body Surface Constraints

Eray Molla, Henrique Galvan Debarba, and Ronan Boulic, Senior Member, IEEE

APPENDIX A

IMPORTANCE $\lambda$ OF A REFERENCE POINT

We define an importance metric, $\lambda$, relying on two properties: proximity, $\lambda_p$, and orthogonality, $\lambda_\perp$ where $\lambda = \lambda_p \lambda_\perp$. It analyzes the surface relative displacement vectors $\mathbf{v}$ that is defined with respect to each element constituting the body surface approximation (Figure 1); these elements are either triangles from the crude meshes approximating the trunk or the head, or limb segment capsules. The proposed metric expresses that nearby surface elements lying perpendicular to the joint are more reliable references to express the joint’s location.

**Proximity ($\lambda_p$):** $\lambda_p$ is inversely proportional to the distance, $||\mathbf{v}||$, hence close surfaces are assigned a higher importance. To avoid numeric instability, we compute the proximity as

$$
\lambda_p = \begin{cases} 
\frac{1}{1 + \epsilon} & ||\mathbf{v}|| \leq \epsilon \\
\frac{1}{||\mathbf{v}||} & ||\mathbf{v}|| > \epsilon 
\end{cases}
$$

(1)

where $\epsilon$ is a small positive number close to zero.

**Orthogonality ($\lambda_\perp$):** We measure the orthogonality $\lambda_\perp$ by exploiting the angle $\alpha$ between the element surface normal $\mathbf{n}$ and $\mathbf{v}$ (Figure 1)

$$
\lambda_\perp = \begin{cases} 
\cos \epsilon \quad \cos \alpha \leq \epsilon \\
\cos \alpha \quad \cos \alpha > \epsilon 
\end{cases}
$$

(2)

Measuring orthogonality is critical for two reasons. First, if the joint projects along the surface normal, it is likely that they are interacting. Second, such interaction context can even carry semantic information in postures such as holding a hand in front of the mouth, eyes, etc.

Fig. 1. The projection of the joint position $\mathbf{p}$ on each element of the body surface approximation produces a reference point, $\mathbf{x}$. Such an element can be a triangle from the crude mesh approximation of the trunk or the head, or a limb segment capsule. In this illustration, we highlight the case where an angle $\alpha$ exists between the surface element normal $\mathbf{n}$ and the surface relative displacement vectors $\mathbf{v}$.

APPENDIX B

HALF SPACE CONSTRAINT RELAXATION

As explained in Section 5.3, we compute two separating planes between a pair of source limb segments, $l_{s_i}$ and $l_{s_j}$, and use them as half-space constraints to preserve the spatial order (Figure 2a). However, enforcing both constraints can over-constrain the problem in case of a different relative orientation between the target segments, $l_{s_i}'$, $l_{s_j}'$, and the source segments (Figure 2b). We make use of a planar interpolation scheme to relax the constraints.

The half-space relaxation for $l_{s_i}'$ works as follows. First, we project $e_{p_{ij}}^+$ onto $l_{s_i}'$, which corresponds to the tangent plane of $l_{s_j}'$ parallel to $e_{p_{ij}}^+$ but pointing at the opposite side. This plane is denoted by $e_{p_{ij}}^+$ (Figure 2c). Then we calculate the signed distances of $l_{s_j}'$ relative to $e_{p_{ij}}^+$ and $e_{p_{ij}}^-$ (Figure 2d).

We compute a weight coefficient for each signed distance by using an S-shaped curve (Figure 3). This continuous curve penalizes the plane causing penetration ($d < 0$)

1. Read as the denormalized egocentric plane of segment $j$ built on the segment $i$. 

Fig. 2. Half-space constraint relaxation scheme. a) The relationship of the segments in the source. b) Possible configuration on the target character. $l_{s_i}'$ violates $e_{p_{ij}}^+$ even though the spatial order is correct. c) $e_{p_{ij}}^-$ is projected on $l_{s_j}'$ and $e_{p_{ij}}^+$ is obtained. d) The signed distance of $l_{s_j}'$ to $e_{p_{ij}}^+$ is negative because of the penetration. e) The relaxed constraint (green plane) is obtained when $e_{p_{ij}}^+$ and $e_{p_{ij}}^-$ are interpolated by $\sigma$. In this specific example, $l_{s_j}'$ does not violate the relaxed half-space constraint.
and encourages the use of the safe one \((d > 0)\). These corresponding weights, \(k'\) and \(k^+\), are converted to an interpolation coefficient as follows:

\[
\sigma = \frac{k'}{k' + k^+}
\]

This coefficient is used to interpolate \(e_p'_{ij}\) and \(e_p^+_{ij}\). As a result, we use that interpolated plane as a relaxed half-space constraint for \(ls'_i\) (Figure 2e).

\[
\text{Fig. 3. Curve used for the signed distance weights } k = \frac{1}{1 + 2^d}
\]

**Appendix C**

**Slerping Standard and Reversed Limb IK Solutions**

This appendix explains how the resulting postures from the standard and reversed limb IK are interpolated. First, we calculate the maximum importance values \(\lambda_i\) of the end and mid joints (respectively \(\max(\lambda^e_{i\text{nd}})\) and \(\max(\lambda^m_{i\text{nd}})\)). We use these values to compute a coefficient \(\gamma\) such that

\[
\gamma = \frac{\max(\lambda^e_{i\text{nd}})}{\max(\lambda^e_{i\text{nd}}) + \max(\lambda^m_{i\text{nd}})}, \quad 0 \leq \gamma \leq 1
\]

We then compute an intermediate limb posture between the standard \((\gamma = 0)\) and the reversed \((\gamma = 1)\) solutions by slerping the state of each joint in the joint angle space according to the coefficient \(\gamma\). In this way,

- if the end joint has close contact while the mid joint doesn’t have, i.e., \(\max(\lambda^e_{i\text{nd}}) \gg \max(\lambda^m_{i\text{nd}})\), \(\gamma\) will be close to zero. Hence, the slerped pose will be close to the standard limb IK solution which considers the end joint as the first priority.
- if the mid joint has close contact while the end joint doesn’t have, i.e., \(\max(\lambda^m_{i\text{nd}}) \gg \max(\lambda^e_{i\text{nd}})\), \(\gamma\) will be close to one. Hence, the slerped pose will be close to the reversed limb IK solution which considers the mid joint as the first priority.
- other combinations will result in compromised poses with continuous transitions.

**Appendix D**

**Storage Cost Analysis**

This appendix analyses the storage cost of a limb joint’s egocentric coordinates and of the egocentric planes.

Expressing the pose of a character with 86 joints requires approximately 1.4 kilobytes by using unit quaternions for rotations and a three dimensional vector for the root position. The cost of egocentric representation of pose on the same character in our implementation is about 21 kilobytes where the body mesh contains 34 triangles. Hence, although our representation significantly increases the memory cost compared to the standard posture representation, its absolute value remains negligible for real-time applications.

As noted in Section 5.1, to express each relative displacement, we store an importance scalar \(\lambda_i\), a 3-DoF reference point \(x_i\), a 3-DoF displacement vector \(v\) and an orientation deviation \(\beta_i\), 4 scalars as a unit quaternion, and the set of \(n\) coefficients \(C_i\) where \(n\) is the length of the kinematic path.

One for the reference primitive and one for the expressed joint, two identifier scalars are held, as well. Therefore, \(1 + 3 + 3 + 4 + n + 2 = 13 + n\) scalars need to be stored to express the relative coordinates of an extremity joint with respect to a single surface primitive.

To consider the relationship with \(m\) surface primitives,

\[
\sum_{r=1}^{m} 13 + n_r = 13m + \sum_{r=1}^{m} n_r \text{ scalars need to be stored, where } n_r \text{ is the length of the kinematic path between the joint and the corresponding reference point, } r. \text{ Given that the length of the longest kinematic path in the body is } N, \text{ e.g., from right hand to left ankle, } O(m(13 + N)) \text{ scalars are required to store the egocentric coordinates of an extremity joint. As a humanoid has four such limbs and the same amount of data needs to be stored twice for each of them, } O(4 \times 2 \times m(13 + N)) = O(8m(13 + N)) \text{ scalar space is used in total.}

For the egocentric planes, we compute a separating plane from each limb segment to all other limbs’ segments. As there are three other limbs and each of them has two segments, the data of six planes are stored per segment. As the same operation is repeated for both segments of each limb, the data of \(4 \times 2 \times 6 = 48\) egocentric planes are stored in total which is considerably less than the storage cost of the egocentric coordinates.

**Appendix E**

**Body size and proportion measurements of the characters used in the study**

Table 1 presents the body measurements of the characters used in the study: Child (C), Alien (A), Woman (W). The first three rows present the absolute measurements for each character in centimeters. The last three rows give the corresponding normalized measurements with respect to the body height. The columns present the data in the following order: body height (H), arm length from the shoulder to the wrist joints (AL), upper arm radius (UAR), lower arm radius (LAR), leg length from the hip to the ankle joints (LL), upper leg radius (ULR), lower leg radius (LLR), belly thickness (BT).

<table>
<thead>
<tr>
<th>Char.</th>
<th>H (cm)</th>
<th>AL</th>
<th>UAR</th>
<th>LAR</th>
<th>LL</th>
<th>ULR</th>
<th>LLR</th>
<th>BT</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (cm)</td>
<td>120</td>
<td>38</td>
<td>5</td>
<td>4</td>
<td>48</td>
<td>6</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>A (cm)</td>
<td>152</td>
<td>42</td>
<td>3</td>
<td>2</td>
<td>66</td>
<td>5</td>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>W (cm)</td>
<td>168</td>
<td>51</td>
<td>5</td>
<td>4</td>
<td>80</td>
<td>8</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>C (%)</td>
<td>100</td>
<td>32</td>
<td>4</td>
<td>3</td>
<td>40</td>
<td>5</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>A (%)</td>
<td>100</td>
<td>28</td>
<td>2</td>
<td>1</td>
<td>43</td>
<td>3</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>W (%)</td>
<td>100</td>
<td>30</td>
<td>3</td>
<td>2</td>
<td>48</td>
<td>5</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 1**

Body measurements of the characters